Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
D		2415494

Dynamic Network Flow Model for Optimal Water Level Management in the Great Lakes

Summary

The Great Lakes constitute a huge and complex dynamic network flow, which is influenced by human regulation and many exogenous environments, and the changes of its water level also affect the neighboring ecology and some stakeholders.

In this paper, we first constructed a network flow model based on the Great Lakes and their related rivers to simulate the flow and transport of water. In addition, we collected historical data of more than ten environmental factors such as evaporation and precipitation of the Great Lakes, established a multifactor influence model, used typical correlation analysis (CCA) to obtain the direct influence matrix of environmental factors on water level changes, combined with Dematel and AISM algorithms to identify the key elements in the system and their degree of influence, and constructed a hierarchy structure of the elements, which was converted to obtain a Great Lake Network diagram of the relationship between water level and its related river flow and 11 environmental factors. Finally, we consider the water level changes with time and season, introduce the seasonal differential autoregressive sliding average model (SARIMA) to dynamize the model, and borrow artificial neural network (ANN) to capture and model the complex nonlinear changes, and finally get the dynamic network flow model of the Great Lakes.

For Problem One, we first determined the optimal water level interval based on historical water level data, determined the ideal water level and water level demand rigidity based on each stakeholder's expectation, and then used interval scanning and iteration to obtain the optimal water level of the Great Lakes at any time of the year.

For problem Two, we established the water level-flow differential equations in the dynamic network flow model of the Great Lakes and added a perturbation factor to improve the model robustness, took the sum of the two-paradigm differences between the optimal water level and the actual water level as the optimization objective, and used genetic algorithms (GA) with multiple iterations to obtain the optimal control method.

For Problem Three, we analyze the local sensitivity of the control algorithm by solving the partial differential and analyze the overall sensitivity by the Sobol index method. Subsequently, the satisfaction quantization function is established, and the calculation learns that the satisfaction index of our control system has improved by 80 percent compared to the satisfaction index in 2017.

For problem Four, we combined the Dematel-AISM static influence model and the SARIMA-ANN dynamic network flow model, used the Sobol exponent method to analyze the sensitivity of the control algorithm to the changes in the environmental conditions, and verified that our control algorithm has strong stability and high robustness.

For Problem Five, we focus on analyzing the special characteristics of Lake Ontario stakeholders and influencing factors based on the dynamic network flow model and control algorithm, and compared with the 2014 plan, our model is better at coping with complex and changing environmental conditions, and our control algorithm can maximize the stabilization of the Great Lakes at the optimal water level.

Keywords: network flow, DEMATEL-AISM analysis, SARIMA-ANN model, GA, Sobol index method

Contents

1 Introduction

The problem revolves around managing the Great Lakes, a vast system of freshwater lakes in the United States and Canada. These lakes have a crucial role in various human and ecological life aspects, including fishing, recreation, power generation, shipping, and more. However, maintaining the appropriate water levels in the lakes is a complex challenge due to various factors: water level regulation, "wicked" problems, complex interactions, etc. Therefore, the Great Lakes water management challenge, including considering the interests of various stakeholders, and finding the optimal water level is very challenging and requires collaboration and cooperation.

Taking into account the background information and constraints identified in the problem statement, we need to address the following problems:

- Determine the most suitable water levels for the Great Lakes throughout the year, considering the diverse interests of stakeholders, each with unique costs and benefits.
- Develop algorithms to regulate optimal water levels in the five lakes based on incoming and outgoing water data.
- Assess the sensitivity of the control algorithms concerning the outflow of the two primary dams. Using 2017 data, evaluate if these new controls would lead to satisfactory or improved water levels for different stakeholders compared to recorded levels.
- Examine how responsive the algorithm is to variations in environmental conditions, such as changes in precipitation, winter snowpack, and occurrences of ice jams.
- Concentrate the in-depth analysis on the stakeholders and factors influencing Lake Ontario, given recent concerns about water level management for this particular lake.

1.1 Our work

Our workflow of this paper is shown in Figure 1 on Page 2.

2 Preparation of the Models

We made the following assumptions to help us build the model. These constructions are the background and basis for our subsequent analysis.

2.1 Assumptions

- Historical data has not been affected by major natural disasters or geological changes.
- The evaporation and precipitation levels of rivers are considered negligible.

2.2 Notations

The key mathematical notations used in this paper are listed in 1.

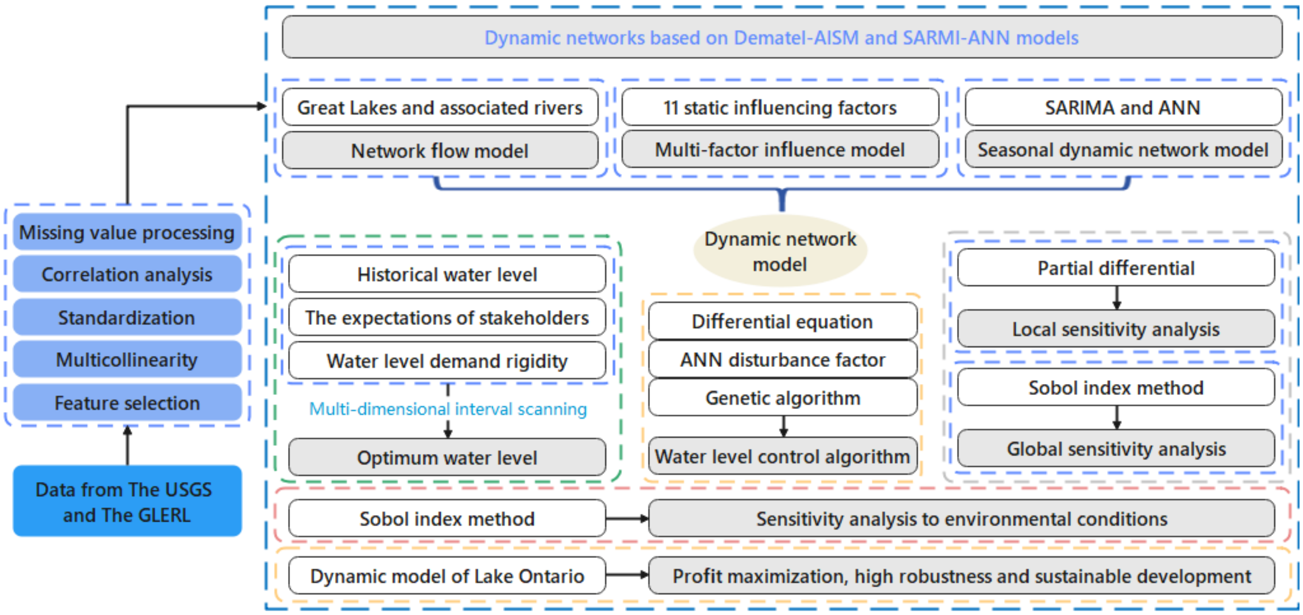

Figure 1: Our work

Table 1: Notations used in this paper

Symbol	Description
x_i	i^{th} factor which influence the water level
$Q_{i,t}$	Lake i^{th} actual level at time t
$Q^*_{i,t}$	Lake i^{th} ideal level at time t
L_i	River i^{th} length
H_i	River i^{th} height
S_i	Lake i^{th} surface area
$I_{i,t}$	Lake i^{th} Inflow at time t
$O_{i,t}$	Lake i^{th} Outflow at time t
$P_{i,t}$	Lake i^{th} precipitation capacity at time t
$E_{i,t}$	Lake i^{th} evaporation capacity at time t
$R_{\alpha\to\alpha+1,t}$	$\alpha\alpha+1$ means River from Lake α to Lake $\alpha+1$, W means the river level, where $\alpha = 1, 2, 3, 4$
$v_{\alpha\to\alpha+1,t,x}$	$\alpha\alpha+1$ means River from Lake α to Lake $\alpha+1$, x means the distance from the point to Lake α, v means the river velocity

3 Core Model

3.1 Data Pre-processing

3.1.1 Data collection

The data for this study were sourced from the following agencies:

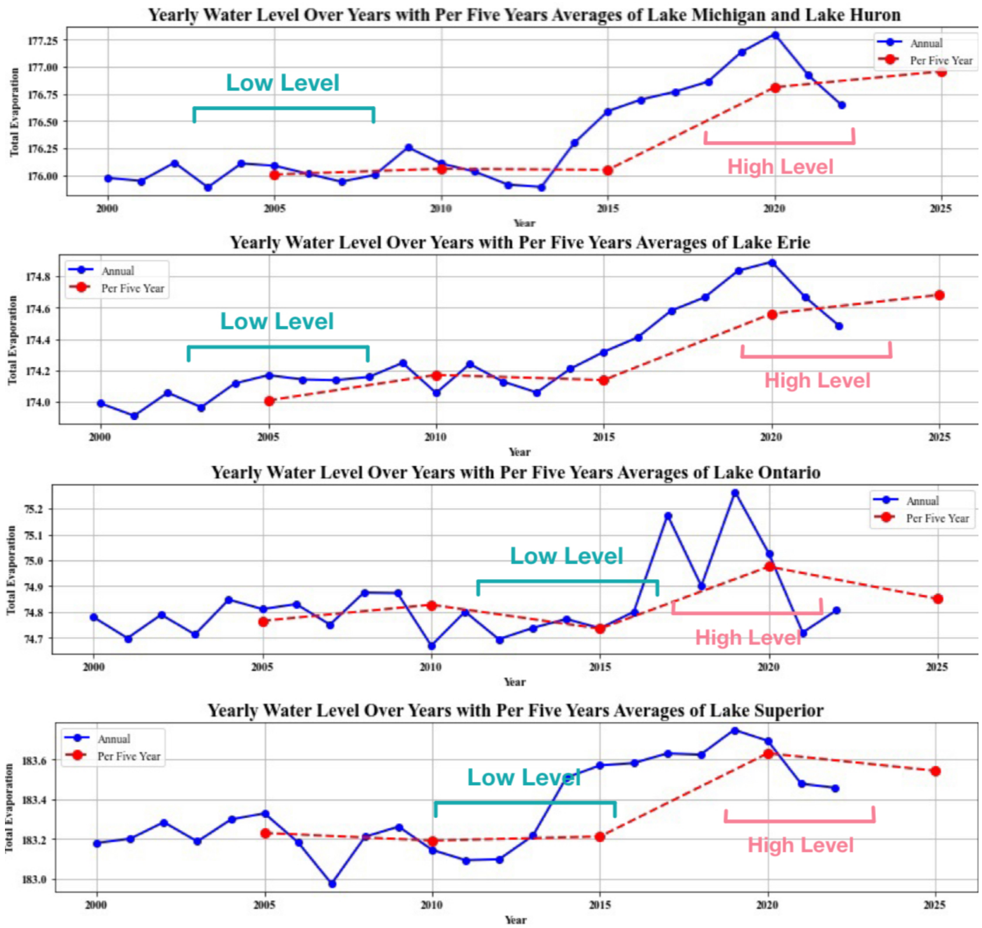

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Figure 2: Yearly mean water level with five Great Lakes

1. Data on surface temperature changes and average precipitation for the Great Lakes from 2000 to 2020 were sourced from NOAA's Great Lakes Environmental Research Laboratory (GLERL), which conducts vital research on the lakes' ecosystems, focusing on the environment.

2. Surface area and flow velocity information were collected from the United States Geological Survey (USGS), an agency dedicated to the study of natural resources and hazards, crucial for informed environmental, resource management, and public safety decisions.

3.1.2 Import Data

In the initial phase of our analysis, we begin by examining the fundamental characteristics of our dataset to gain a comprehensive understanding of its structure. This includes identifying the column names and discerning the data types contained within. To further our understanding, we delve into descriptive statistical analysis, encompassing metrics such as the mean, median, and standard deviation. These measures are essential for providing a preliminary overview of the dataset's central tendencies and variability:

Subsequently, we present visual representations to illustrate the linear trends in water levels and associated factors over time. These graphical depictions are crucial for intuitively understanding the progression and interrelationships of the variables under study. They serve not only as a foundation for our further analysis but also as a means to visually communicate the patterns and trends observed in the data. In the preliminary stage of data exploration regarding factors influencing water level changes in the Great Lakes, we employed correlation analysis and exploratory data analysis (EDA). This analysis revealed that environmental variables such as evapotranspiration, precipitation, surface water temperature, and flow velocity exhibit strong correlations with fluctuations in water levels. These initial observations suggest a significant role for these environmental factors in the water level regulation mechanism, where their trends markedly influence the dynamic equilibrium of water levels.

Consequently, our study has chosen to focus on rounded precipitation, surface temperature, and flow velocity as the primary variables for further investigation. Building on these theoretical underpinnings and empirical observations, our study will utilize mathematical modeling and machine-learning techniques to quantitatively analyze these selected variables. We aim to elucidate the intricate interplay between these factors and water level changes, and to develop a predictive model for forecasting future trends in water levels.

3.2 Building Core Model

The management of water levels in the Great Lakes is a complex task that necessitates a careful balance among various demands, including flood control, navigation, hydropower generation, recreation, and ecological preservation. The Great Lakes, along with their tributary rivers, form a unidirectional network graph that transitions from point to line. To effectively simulate the water flow between the lakes and their transmission through rivers, we employ a network flow model. This model conceptualizes each lake and connecting river as a node, with the direction of water flow represented as edges, thereby providing a clear depiction of the system's flow dynamics.

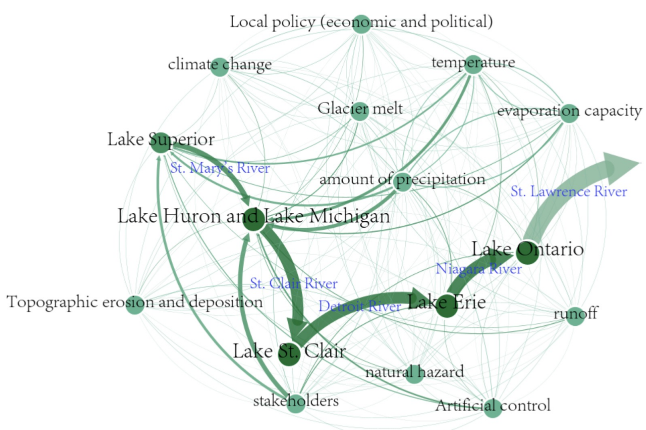
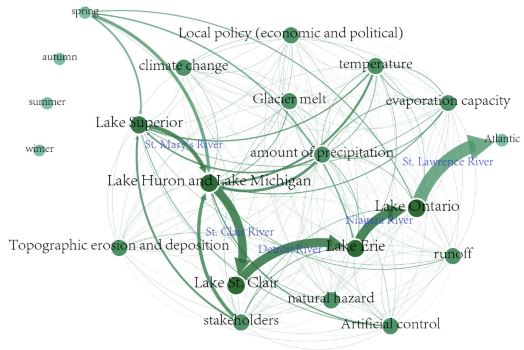


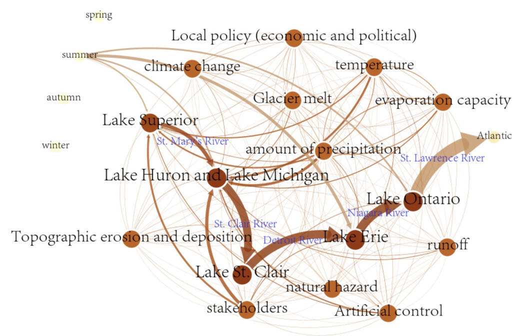
Figure 3: Static model

The network flow model is not only instrumental in illustrating the water movement within the system but also proves vital in analyzing the sensitivity of the control algorithm to environmental changes. Factors such as precipitation, winter snowfall, and ice closures can significantly impact the system, and understanding their effects is crucial for ensuring the model's effectiveness and robustness across different scenarios.

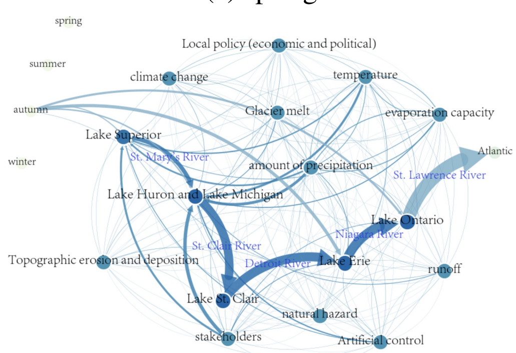
In line with these objectives, we have developed a static model of the Great Lakes and their associated rivers based on a directed network flow approach. To better visualize and understand this complex system, we utilized Gephi, a sophisticated network visualization tool, to create the following figure. This visual representation not only aids in comprehending the model but also assists in identifying key areas for analysis and intervention in water level management:



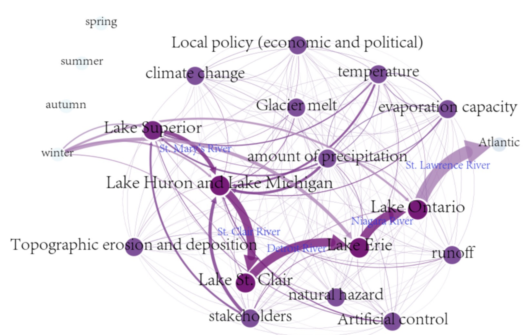
(a) spring



(b) summer



(c) autumn



(d) winter

In response to the complexities of climate change, our model aims to improve prediction accuracy by considering a wider array of influential factors crucial for understanding water level trends and seasonal variations. We've gathered historical data on temperature, precipitation, evapotranspiration, and other external factors. Integrating these into our network flow model of the Great Lakes and their rivers, we've constructed a multifactor influence model. This approach enhances our understanding of the dynamics driving water levels and volumes, enabling more accurate simulations and predictions of the system's behavior under various environmental conditions.

In our study, we employ the Decision Making Trial and Evaluation Laboratory (DEMATEL) method to decipher complex causal relationships between factors such as temperature, precipitation, and evaporation, and their impact on water levels and volumes. This method effectively calculates and visually represents the influence and impact of these factors, aiding researchers and decision-makers in understanding their roles and interrelationships within the system. However, DEMATEL has limitations in identifying the hierarchical structure of these factors. To address this, we integrate the Aggregation and Interpretative Structural Modeling (AISM) method. AISM breaks down the complex system into sub-systems or elements and constructs a multilevel hierarchical structure using Boolean algebra operations. This approach enables us to analyze direct binary correlations between the subsystems, forming a layered topological map. While AISM excels at structuring these elements hierarchically, it does not quantify their influence on the system. By innovatively combining DEMATEL and AISM, our network flow and influence model not only identifies key elements in the water level control system and their degrees of influence but also constructs a comprehensive hierarchical structure of these elements. This combined approach enhances our ability to understand and predict the behavior of the water level control system in a more nuanced and structured manner.

We began our analysis by applying Canonical Correlation Analysis (CAA) to examine the relationships between two variable sets: environmental factors and water level data. Our objective was to identify linear combinations within these sets that maximized their mutual correlation. The principle is as follows:

Given the linear combinations:

$$\begin{aligned} U &= a_1 x_1 + a_2 x_2 + \cdots + a_p x_p = \mathbf{a}'\mathbf{X} \\ V &= b_1 y_1 + b_2 y_2 + \cdots + b_q y_q = \mathbf{b}'\mathbf{Y} \end{aligned}$$

Where:

- U and V are the canonical variables.
- $\mathbf{X}$ and $\mathbf{Y}$ are vectors of the original variables in the two sets being analyzed.
- $\mathbf{a}$ and $\mathbf{b}$ are vectors of the coefficients for the linear combinations that form the canonical variables.
- a_i and b_j are the coefficients corresponding to the i^{th} and j^{th} variables in the $\mathbf{X}$ and $\mathbf{Y}$ sets, respectively.

The variances and covariance are given by:

$$\begin{aligned} \mathrm{Var}(U) &= \mathbf{a}'\mathrm{cov}(\mathbf{X}, \mathbf{X})\mathbf{a} = \mathbf{a}'\Sigma_{11}\mathbf{a} \\ \mathrm{Var}(V) &= \mathbf{b}'\mathrm{cov}(\mathbf{Y}, \mathbf{Y})\mathbf{b} = \mathbf{b}'\Sigma_{22}\mathbf{b} \\ \mathrm{cov}(U, V) &= \mathbf{a}'\mathrm{cov}(\mathbf{X}, \mathbf{Y})\mathbf{b} = \mathbf{a}'\Sigma_{12}\mathbf{b} \end{aligned}$$

Where:

- $\mathrm{Var}(U)$ and $\mathrm{Var}(V)$ are the variances of the canonical variables.
- $\mathrm{cov}(U, V)$ is the covariance between the canonical variables.
- Σ_{11} and Σ_{22} are the covariance matrices of $\mathbf{X}$ with itself and $\mathbf{Y}$ with itself, respectively.
- Σ_{12} is the covariance matrix between $\mathbf{X}$ and $\mathbf{Y}$.

The correlation between U and V is defined as:

$$\rho = \mathrm{corr}(U, V) = \frac{\mathbf{a}'\Sigma_{12}\mathbf{b}}{\sqrt{\mathbf{a}'\Sigma_{11}\mathbf{a}\mathbf{b}'\Sigma_{22}\mathbf{b}}}$$

Where ρ is the canonical correlation coefficient.
The objective of CCA is to maximize this correlation:

$$\max \rho = \frac{\mathbf{a}'\Sigma_{12}\mathbf{b}}{\sqrt{\mathbf{a}'\Sigma_{11}\mathbf{a}\mathbf{b}'\Sigma_{22}\mathbf{b}}}$$

Following this, we conducted multivariate regression analyses to quantify the contribution of each factor to water level changes. This process yielded a direct impact matrix, providing insights into the extent of influence each environmental factor has on water level variations. This two-step analytical approach allowed us to not only understand the interconnections between variables but also to measure their impacts on the Great Lakes' water levels

$$A = \begin{bmatrix} 0 & a_{12} & \dots & a_{1n} \\ a_{21} & 0 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & 0 \end{bmatrix} \tag{1}$$

Then we need to do the normalization(we complete the data-processing part):

$$B = \frac{a_{ij}}{max(\sum_{j=1}^{n} a_{ij})} \tag{2}$$

To ensure the reliability of our direct impact matrix, we conducted diagnostic tests focusing on two key aspects. First, we examined multicollinearity to confirm that no high correlations exist among the independent variables. High multicollinearity would lead to negligible elements in the matrix, undermining the validity of our findings. Second, we implemented regularization to manage the model's complexity, thereby avoiding overfitting. For this purpose, ridge regression was employed, adding a penalty term to the loss function. This approach effectively balances the model's fit to the data with its simplicity, ensuring both the accuracy and generalizability of our predictive insights.

In the subsequent phase of our analysis, we computed a combined influence matrix. This matrix was instrumental in determining the degree of influence and centrality among the various factors. We then assessed the positivity and negativity of each indicator, crucial for understanding their respective impacts on water levels. Given our focus on comparing the relative influence of these factors on water levels, we proceeded to establish a partial order of the data. This ordering served as a foundation for applying Aggregation and Interpretative Structural Modeling (AISM) operations. Through these operations, we successfully generated an antagonistic hierarchical topology map. This map visually represents the complex interplay and hierarchical relationships between the factors, providing a clear depiction of their relative influence in the system:

$$T = (B + B^2 + \cdots + B^k) = \sum_{k=1}^{\infty} B^k = B(I - B)^{-1} \tag{3}$$

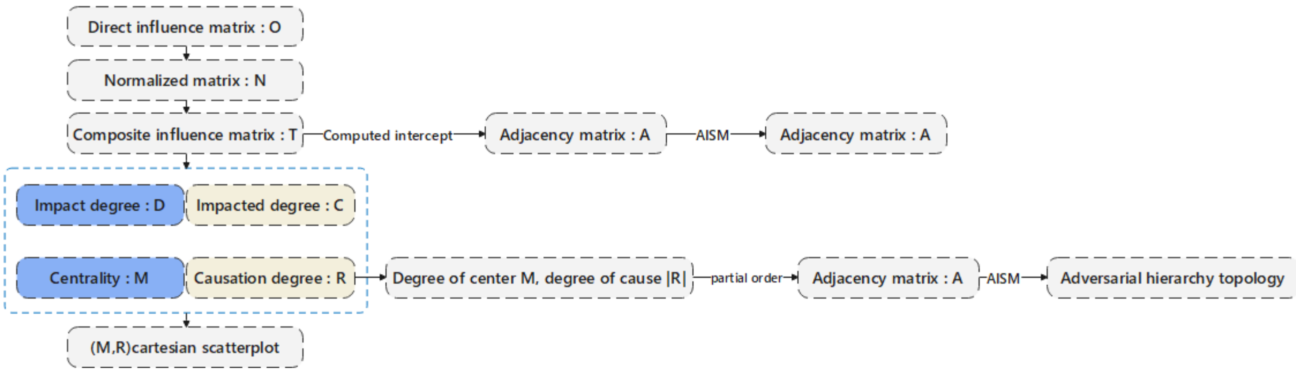


Figure 5: Process of AISM

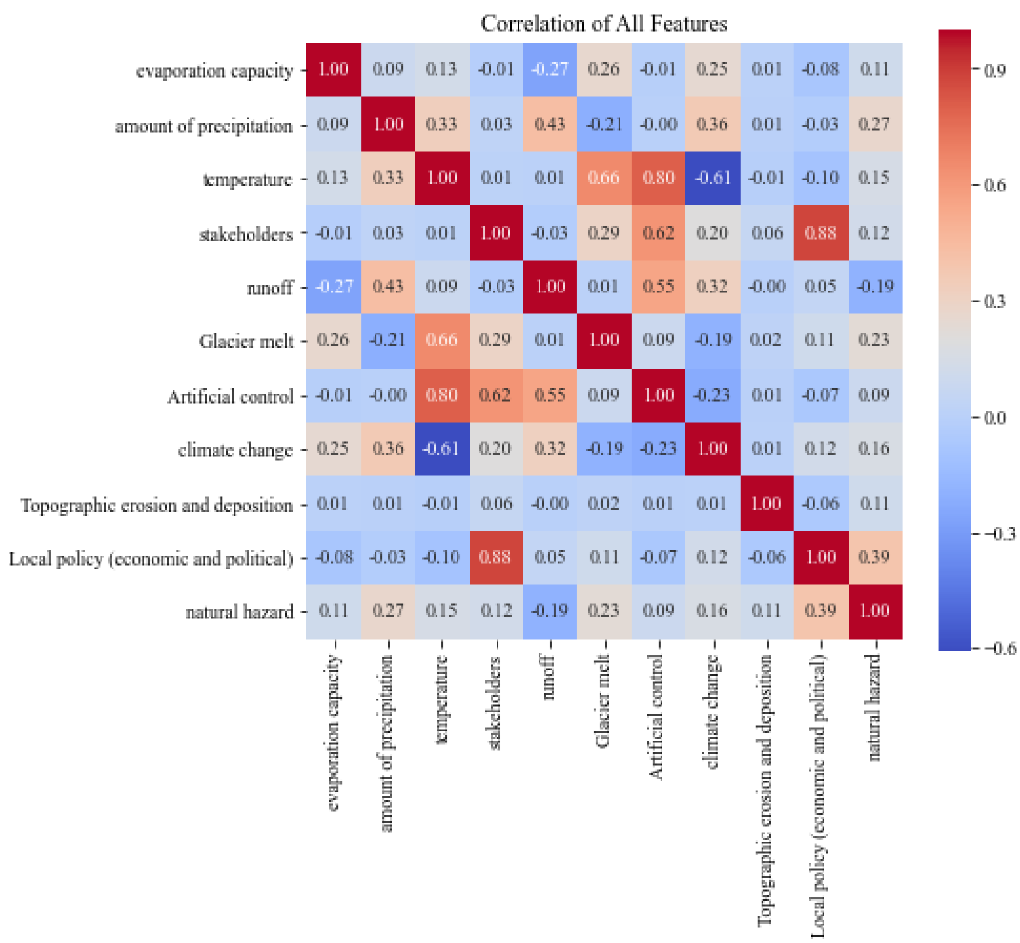


Figure 6: Correlation Heatmap of All Features

After the DEMATEL-AISM model process, we obtain the heatmap correlation of eleven attributes, the heatmap as shown above.

To enhance our static model and account for temporal variations in water levels, including seasonal changes and impacts from factors like glacier snowmelt, we recognized the limitations of solely relying on time series models or adding numerous factors, which could lead to significant errors. We also noted that while neural networks provide a better fit than linear models, they are prone to overfitting due to data noise and limited sample sizes. To address these challenges, we opted to use SARIMA (Seasonal Autoregressive Integrated Moving Average) for processing and predicting time series data. Integrating SARIMA into our dynamic network model enables more accurate simulation and prediction of the Great Lakes' and related rivers' water level trends over time, including capturing seasonal pattern changes.

For the model's nonlinear aspects, we employed Artificial Neural Networks (ANN) to model complex nonlinear relationships. ANN's capability to automatically adjust model parameters based on new data from time series analysis ensures ongoing accuracy and relevance. By combining SARIMA and ANN, our dynamic network model's generalization ability is significantly enhanced. This combination allows the model to handle a broader range of conditions and more complex scenarios, including simulating the impacts of extreme events like floods or droughts. This approach is crucial for formulating effective contingency plans in response to these hydrological challenges.

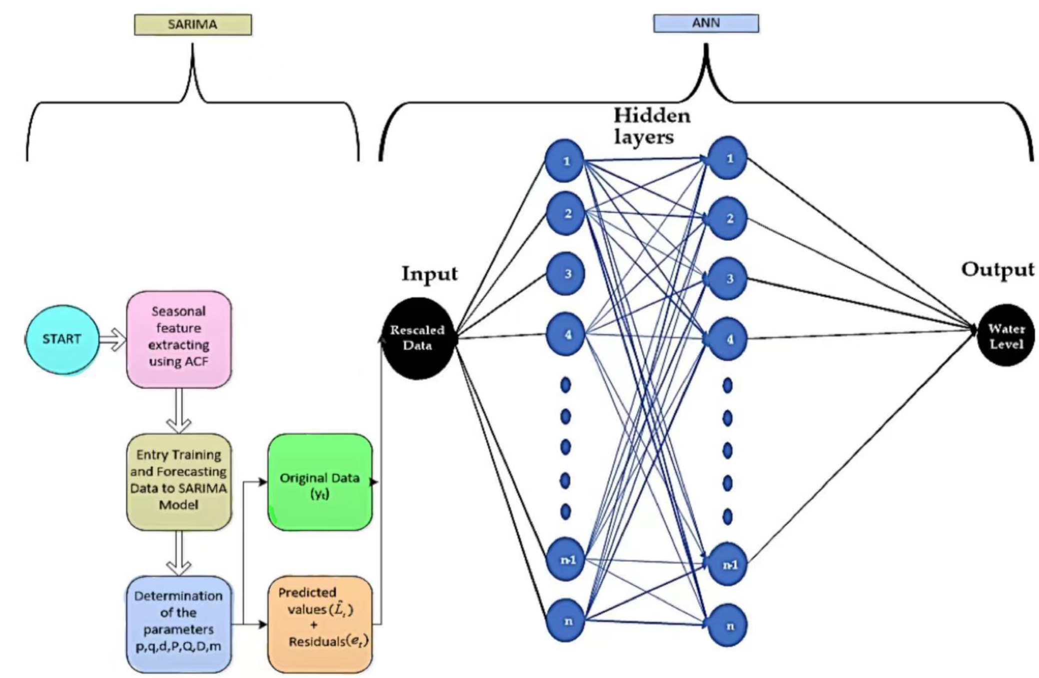


Figure 7: Process of ANN

The SARIMA process decomposes the trend and seasonal influence of mean water level, and we obtain the graph of the influence below.

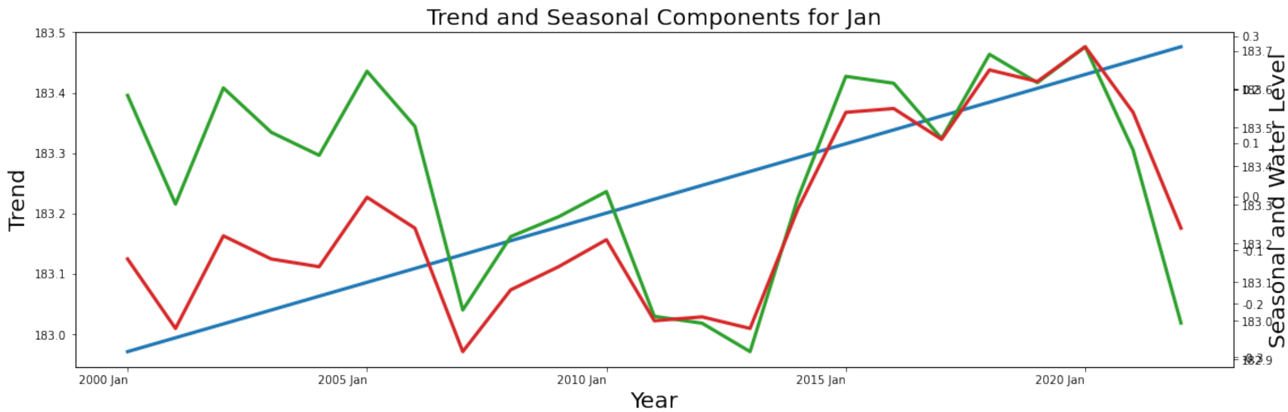


Figure 8: Trend and Seasonal Components for January Data

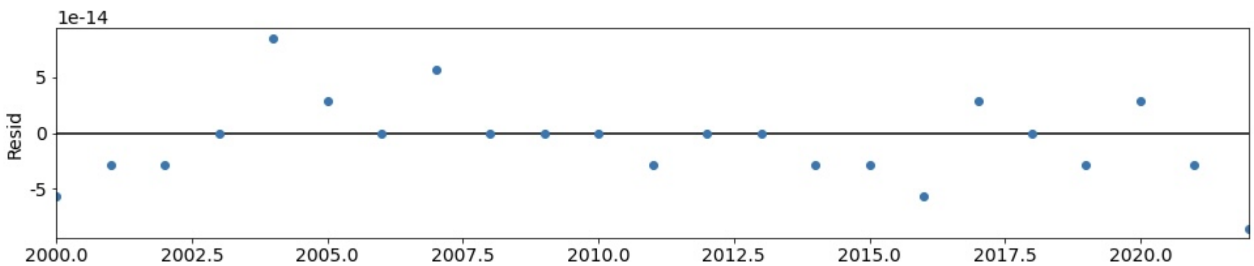


Figure 9: Residual for January Data

We commenced our analysis by dissecting the water level data into seasonal, trend, and stochastic components using STL (Seasonal-Trend decomposition using Loess) time series decomposition. This crucial step helped in uncovering the seasonal patterns and trends in water level changes, guiding the selection of the seasonal cycle parameter s. Next, we carefully chose the SARIMA model parameters, including both seasonal (P,D,Q) and non-seasonal (p,d,q) components, informed by our analysis of seasonality. The seasonal cycle parameter s was determined with the support of ACF (Autocorrelation Function) and PACF (Partial Autocorrelation Function) plots. We trained the SARIMA model on historical water level data, using cross-validation and validation set splitting for performance evaluation. The trained SARIMA model was then utilized to predict water level changes post-2022, providing future water level forecasts and their confidence intervals. This serves as a reference for optimal water level ranges. Given that our linear model focused on the three most correlated factors, we analyzed SARIMA model residuals to identify non-random patterns. Discovering that these residuals held predictable elements, we employed ANN (Artificial Neural Networks) to model and regress these residuals, capturing influences beyond the primary time series model and further refining our forecast accuracy. By merging the predictions from the SARIMA model and the ANN residual analysis, we obtained a corrected water level forecast. This forecast, integrated with multi-dimensional stakeholder demand and dependency analysis, furnishes a scientific basis and decision-making support for the effective management of water levels in the Great Lakes.

The SARIMA model equation is given by:

$$\Phi_P(L^s)\phi_p(L)(1-L)^d(1-L^s)^D y_t = \Theta_Q(L^s)\theta_q(L)\epsilon_t \tag{4}$$

where:

- y_t is the time series observation.
- L is the lag operator, $L^k y_t = y_{t-k}$.
- $\phi_p(L)$ and $\Phi_P(L^S)$ are the autoregressive operators of non-seasonal and seasonal parts, respectively, defined as $1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$ and $1 - \Phi_1 L^S - \Phi_2 L^{2S} - \cdots - \Phi_P L^{PS}$.
- $\theta_q(L)$ and $\Theta_Q(L^S)$ are the moving-average operators of non-seasonal and seasonal parts, respectively, defined as $1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$ and $1 + \Theta_1 L^S + \Theta_2 L^{2S} + \cdots + \Theta_Q L^{QS}$.
- ε_t is the error term.

4 Task1: Optimizing Year-Round Water Levels in the Great Lakes for Diverse Stakeholder Benefits

4.1 Background and analysis of the problem

As climate change and human impacts intensify, managing the water levels of the Great Lakes—Superior, Michigan, Huron, Erie, and Ontario—becomes crucial due to their role in ecosystem health, economic activities, and water availability. However, fluctuating water levels pose risks like floods, water quality issues, and biodiversity loss.

Our study seeks to find the optimal water levels for balancing ecological, economic, and social needs, considering the diverse interests of stakeholders like environmental groups, local governments, and businesses. Using a combination of factor influence analysis (Dematel-aism model), time series analysis (SARIMA model), and deep learning (ANN model), we aim to predict water level changes and understand stakeholder dependencies.

The goal is to develop a comprehensive model that integrates historical data and stakeholder preferences to recommend sustainable water level management practices. This research will inform policymakers and communities, offering a scientific and practical framework for effective and equitable water level management in the Great Lakes region.

4.2 Model description

This study focuses on determining the optimal water level at any given time throughout the year by leveraging historical data spanning from 2000 to 2022. Considering the critical influence of seasonal variations and stakeholders' varied interests, the research adopts a monthly resolution to identify optimal water levels.

Acknowledging the practical constraints associated with stakeholder diversity and influence, the study employs a methodology that integrates historical feasible equilibrium water levels within each month with a dependency scan approach. The objective is to derive a balanced solution that aligns with stakeholders' needs and maintains a historically verified water level range, accommodating their dynamic requirements.

The research posits that the relatively stable fluctuations in the Great Lakes' water levels during the study period indicate a controllable regulation within an acceptable range. This stable range, rooted in historical experience, reflects a harmonious equilibrium that balances ecological preservation and economic development. Thus, the study proposes a methodology for obtaining and sustaining this historical water level range, ensuring a pragmatic and sustainable approach to water level management.

For the given historical lake levels F_{ij}, where i is the month and j the year, the optimal range for month i is determined as $[\min(F_{i1}, ..., F_{in}), \max(F_{i1}, ..., F_{in})]$. This interval is historically accepted as a balanced level, suitable for diverse stakeholder interests. Relying on this historical data is prudent for modeling, given the continuity in lake level management and inherent forecasting uncertainties.

Stakeholders' preferences for water levels vary, thus individual optimal levels within this historical range must be ascertained. Our model utilizes the maximum, median, and minimum values of the historical interval to represent high, moderate, and low water level demands, respectively. For example, stakeholders requiring high water levels may be characterized by the historical maximum, set as $\max(F_{i1}, ..., F_{in})$, while those needing moderate stability are initialized at the mean, $\mathrm{mean}(F_{i1}, ..., F_{in})$.

Considering stakeholders' needs extend beyond water levels to variables such as flow rates and temperatures, the core model's Dematel-TAISM analysis provides correlation coefficients, formulating the water level relationship as:

$$\text{water\_level} = a \cdot \text{temp} + b \cdot \text{prep} + c \cdot \text{evapo} + \dots$$

The desired direction of water level change for each stakeholder (increase or decrease) is denoted by ± 1, which when multiplied by the respective correlation coefficients and summed, yields the influence sum w, indicative of dependency.

The weight of asset i is calculated as:

$$w_i = \sum_{j=1}^{n} (r_{ij} \times d_{ij})$$

where:

- w_i denotes the weight of asset i.
- r_{ij} denotes the return of asset i in scenario j.
- d_{ij} denotes the decision variable for asset i in scenario j.

The sensitivity coefficients for each stakeholder, denoted as $\{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_m\}$, where m is the total number of stakeholders, are obtained through the proposed methodology. In consideration of real-world scenarios, stakeholders with higher sensitivity coefficients exhibit a greater dependence on water levels, implying a more rigid response to fluctuations. To quantify the adjustment range of acceptable water level fluctuations for each stakeholder, the sensitivity coefficients are normalized and reciprocated, yielding adjustment factors $\{w_1, w_2, w_3, \dots, w_m\}$.

This dependency calculation approach provides a quantitative reference for water level management, facilitating a better understanding of stakeholders' sensitivity and demands regarding water level variations. It aids in formulating comprehensive and balanced water level management strategies, ensuring a win-win situation among various stakeholders in the management of the Great Lakes water levels.

Once the optimal acceptable water level range and adjustment weights for each stakeholder are established, for every stakeholder, based on their dependency coefficient (α), each adjustment to the water level is calculated as

$$Q_{i,k} = Q_{i,k-1} + w_{i,k} \times Q_{i,k} \times 0.01$$

where:

- $Q_{i,k}$ denotes the i^{th} stakeholder's water level in k^{th} iteration.
- $W_{i,k}$ denotes the i^{th} stakeholder's weight in k^{th} iteration.

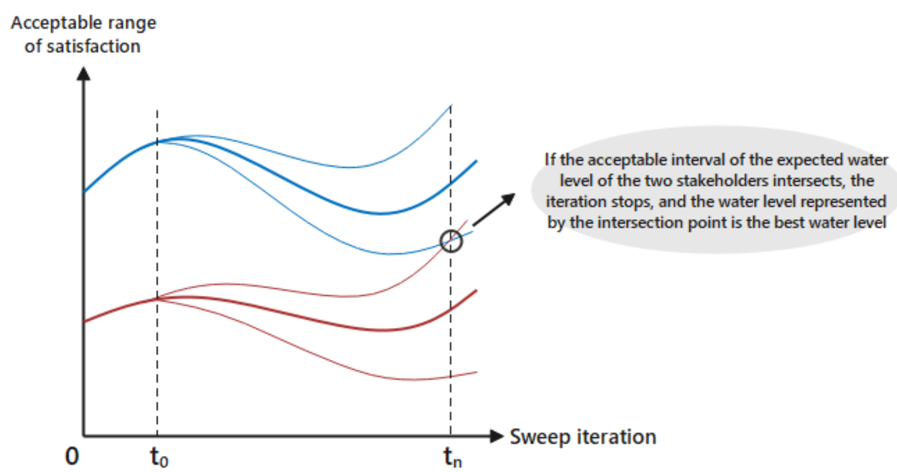


Figure 10: Optimal Water Level Sweeping Process

Multiple stakeholders simultaneously conduct several rounds of water level adjustments until a convergence point in a multi-dimensional space is found where all stakeholders' interests overlap. $F_{ik}(n^*)$ represents this point of convergence, indicating the final balance of interest demands and hence the optimal water level for that month. Specifically, we take into account the adjustment of the median initial value. For stakeholders whose needs are for stable, moderate water levels, the rigidity of their demands could influence the final optimal water level decision-making. Once the interval convergence is achieved, considering the final values of stakeholders with median demands can help realize a more broadly accepted balance point. Moreover, our study employs a fine step size in the iterative scanning process, enabling a precise balance of different stakeholders' needs, especially those who are sensitive to water level changes.

Considering the evolution of the times, climate change, and macro adjustments to government policies, the aforementioned model should be adjusted to a dynamically tunable one. First, using the SARIMA+ANN algorithm from the core model, we predict the water levels for the next 1 to p years. Then, returning to the initial steps of this task's algorithm, we recalculate the future optimal water level feasible interval $[\min(Q_{i1}, ..., Q_{in}), \max(Q_{i1}, ..., Q_{in})]$, and through a similar algorithm, compute the dynamic optimal water level $Q_{ik}(n^*)$ for a given month in a given year. By integrating the SARIMA model and ANN algorithms, this analysis process not only accurately captures the seasonality and long-term trends of the Great Lakes' water levels but also effectively considers other potential influencing factors, thereby enhancing the precision and robustness of water level forecasts. This provides strong support for developing scientifically based water level management strategies for the Great Lakes.

5 Task2: Algorithmic Regulation of Optimal Water Levels in the Five Great Lakes

5.1 Background and analysis of the problem

In our research, we've achieved a significant milestone by determining optimal monthly water levels that synthesize the interests of various stakeholder groups. The next step involves refining our algorithm to manage each of the Great Lakes according to these optimal levels. This is crucial as the water level of each lake is integral to its ecological sustainability and balance.

Our objective is to develop an intelligent algorithm tailored to the unique conditions and geography of each of the five lakes, incorporating their historical inflow and outflow data. This approach aims to maintain the optimal water levels in each lake, ensuring a harmonized balance across the entire system.

Given the dynamic nature of hydrological systems, with continuous state changes over time, we propose using a differential equation model. This model is adept at capturing the dynamic characteristics of water level changes, accounting for various influencing factors like rainfall and evaporation. It describes the system's evolution through time derivatives of state variables (e.g., water level, flow rate), providing a mathematical representation that closely mirrors the physical processes.

To enhance the model's accuracy, we will establish differential equations for the water levels of the Great Lakes and their associated river flows, forming the constraints of our optimization problem. Additionally, we acknowledge the presence of an unexplained perturbation term , in the water levels, which we will incorporate into the model. This will allow for a more robust and realistic representation of the system, ensuring that our algorithm can effectively manage the water levels under a variety of conditions and scenarios:

$$\frac{dQ_{i,t}}{dt} = w_{i,t} \times I_t - w_{o,t} \times O_t + w_{p,t} \times P_t - w_{e,t} \times E_t + \epsilon_t, \quad i = 1, 2, \ldots, 5$$

where:

$I_{i,t}$ denotes Lake i^{th} Inflow at time t

$O_{i,t}$ denotes Lake i^{th} Outflow at time t

$P_{i,t}$ denotes Lake i^{th} precipitation capacity at time t

$E_{i,t}$ denotes Lake i^{th} evaporation capacity at time t

ϵ_t denotes the part in $Q_{i,t}$which can't be explained by time series model

In managing the Great Lakes' water levels, it's essential to adhere to the water conservation equation. This principle ensures that the outflow from upstream lakes matches the inflow into rivers and that river outflows equal inflows into downstream lakes. Integrating this equation into our model helps maintain the natural hydrological balance and supports sustainable water level management across the Great Lakes system.

$$Q_{i,t} = R_{i \to i+1,t}, \quad i = 1, 2, \ldots, 5$$

$$R_{i\to i+1,t} = Q_{i+1,t} \quad i = 1, 2, \ldots, 5$$

where:

$$R_{i\to i+1,t} \quad \text{denotes the level of river from } i^{th} \text{ Lake to } (i+1)^{th} \text{ Lake}$$

Of course, the water level receives physical conditions and the height of the river, except that the Atlantic Ocean does not need to take into account changes in the water level, and there is no limitation on the water level:

$$Q_{i,t} \leq H_i \quad i = 1, 2, \ldots, 5$$

where:

$$Q_6 \quad \text{denotes the Atlantic ocean so that the level can be ignored}$$
$$H_i \quad \text{denotes the height of the water course}$$

Although we determined the optimal yearly water levels for the Great Lakes, actual levels naturally fluctuate over time due to unforeseen factors. To address these deviations, we propose a multidimensional nonlinear optimization model. The objective function of this model is defined as the sum of the squared differences between the combined ideal and actual water levels of the stakeholders. Specifically, we use the sum of the Euclidean distances between the actual water levels of the five lakes and their ideal levels at any given time t as our optimization problem's objective function. This approach aims to minimize the deviation from optimal levels, accommodating the dynamic and unpredictable nature of the Great Lakes' water system

$$\text{Minimize:} \quad -\sum_{t=1}^{12} \left\| Q_{i,t} - Q'_{i,t} \right\|_2^2$$

where:

$$Q_{i,t} \quad \text{denotes the Lake } i^{th} \text{ actual level at time t}$$
$$Q^*_{i,t} \quad \text{denotes the Lake } i^{th} \text{ ideal level at time t}$$

In our differential equation model, we incorporate a range of variables and parameters, such as climatic factors and terrain features, and introduce perturbation factors. To address the complex, nonlinear, and challenging-to-quantify variables, we integrate the principles of ANN (Artificial Neural Networks). This integration allows us to quantify these variables effectively and elucidate their interactions, laying the groundwork for comprehensive analysis and enhancing the model's complexity and applicability.

$$\begin{aligned}
\text{Minimize:} \quad & -\sum_{t=1}^{12} \left\| Q_{i,t} - Q'_{i,t} \right\|_2^2 \\
\text{Subject to:} \quad & \frac{dQ_{i,t}}{dt} = w_{i,t} \times I_t - w_{o,t} \times O_t + w_{p,t} \times P_t - w_{e,t} \times E_t + \epsilon_t, \quad i = 1, 2, \ldots, 5 \\
& Q_{i,t} = R_{i\to i+1,t}, \quad i = 1, 2, \ldots, 5 \\
& \frac{\partial R_{i\to i+1,t}}{\partial t} + \frac{\partial (R_{i\to i+1,t} v_{i\to i+1,t,x})}{\partial s_i} = F(t), \quad i = 1, 2, \ldots, 5 \\
& R_{i\to i+1,t} = Q_{i+1,t} \quad i = 1, 2, \ldots, 5 \\
& Q_{i,t} \leq H_i \quad i = 1, 2, \ldots, 5
\end{aligned}$$

where:

$F(t)$ is the source term caused by changes in lake level and outflow, and this source term can be expressed in terms of changes in lake level and outflow

$v_{i\to i+1,t}$ is the velocity of river from Lake i to Lake i+1

s_i denotes the distance from i^{th} Lake

For solving the optimization problem, we recognize the effectiveness of genetic algorithms as a global optimization technique. Capable of finding optimal solutions in multi-dimensional spaces for complex objective functions, genetic algorithms are particularly suited for non-convex functions or scenarios with multiple local optima. Their problem-agnostic nature and adaptability to various constraints and nonlinear relationships significantly improve the generality and robustness of our dynamic network model. Therefore, we employ adaptive genetic algorithms to seek the optimal solution set for our optimization problem, ensuring an effective and robust approach to managing the Great Lakes' water levels.

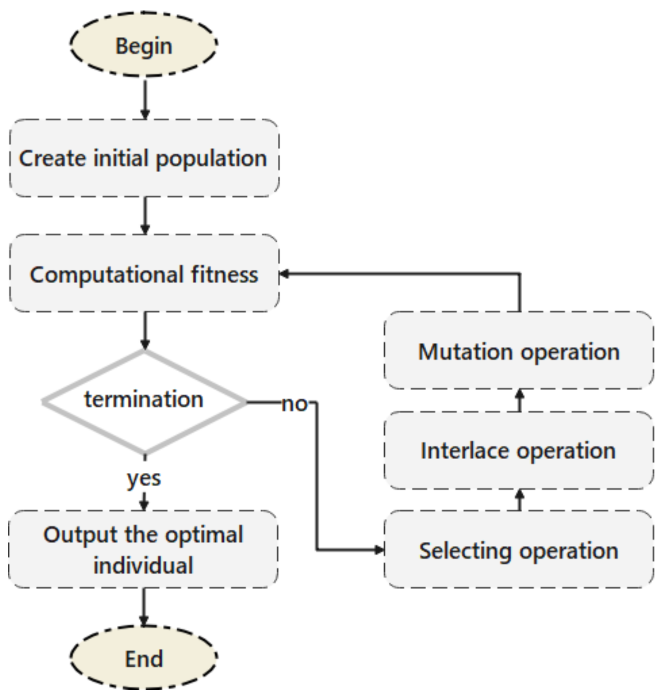


Figure 11: Genetic Algorithm Process

Genetic algorithms are particularly adept at solving complex water level control problems due to their multi-objective optimization capabilities. They effectively balance the diverse and often conflicting needs of environmental protection, shipping, and residential environments. These algorithms excel in handling the complex nonlinear relationships between various factors like temperature, wind speed, and precipitation, without being constrained by linear or convex assumptions.

Furthermore, their adaptive nature allows for continuous adjustment to dynamic environmental changes, iteratively evolving to find the most suitable water level control strategy. By exploring multiple potential solutions simultaneously, genetic algorithms efficiently navigate toward the global optimum, considering a variety of water level control strategies from the onset.

Overall, the genetic algorithm's strengths in multi-objective optimization, adaptability to dynamic changes, and parallel exploration of solutions make it an ideal choice for addressing the complex challenges of water level management in the Great Lakes.

5.2 Model description

1. The fitness function in our genetic algorithm is designed to address multi-objective optimization requirements, such as balancing the needs of environmental protection with human activities. It calculates the actual water levels for each strategy, utilizing differential equations to simulate water level changes. The fitness function is formulated as Minimize: $-\sum_{t=1}^{12}\left\|Q_{i,t}-Q'_{i,t}\right\|_2^2$, incorporating various factors like temperature, wind speed, and tides, and their nonlinear relationships.

2. Termination Condition: We have defined the termination condition for our algorithm as the point where changes in the fitness function are less than 1 percent. Once this criterion is met, the algorithm ceases iterations, ensuring an efficient and effective optimization process.

6 Task3: Evaluating Control Algorithm Sensitivity for Dam Outflows Against 2017 Data

6.1 Background and Analysis of the problem

Our research aims to evaluate and refine control algorithms for dam outflow management, improving water level regulation by using 2017 water level data to benchmark performance and stakeholder satisfaction. The research involves a sensitivity analysis of current algorithms to pinpoint influential factors and parameters, comparing the efficacy of existing and proposed strategies across stakeholder needs like flood control and water supply.

Using sensitivity analysis models, the research applies partial differentiation and Sobol indices for local and global sensitivity assessments, creating quantifiable satisfaction metrics for stakeholders. The anticipated result is the development of enhanced algorithms that outperform the 2017 baseline in efficiency and responsiveness, thus supporting sustainable water resource management for various stakeholders.

6.2 Partial Differentiation Method:Local Sensitivity Analysis

The partial differentiation method allows us to quantify the specific impact of each control dam on water level changes, providing an intuitive understanding of the system's response. This helps us assess the sensitivity of each control dam, which is instrumental in optimizing control strategies and allocating adjustment measures for more efficient water level management. Therefore, based on the differential equations obtained in the second question and leveraging the network model from the first question, we use the partial differentiation method to analyze local sensitivity, specifically examining the sensitivity of each control dam.

By discretizing the partial differential equations governing the water inflow and outflow for each dam at finite difference points, we approximate the solution to the original continuous problem. This approach allows us to obtain discrete data values that approximate the sensitivity of various factors related to the dams' influence on water level control.

The finite difference method is particularly suitable for our research topic, given its ability to analyze the dynamic relationship between dam factors and water level, handle discrete and complex correlation data characteristics, and accurately assess the rates of influence. This approach enhances the model's interpretability and feasibility.

To begin, we have two control dams, with one held constant at its optimal water level state while adhering to control variable rules. We observe changes in water level to calculate partial derivatives:

$$Y = f(x_1, x_2, \ldots, x_n)$$

calculating partial derivatives to assess the degree of change in Y when the parameters X_i undergo small variations.

We will solve the form of the partial differential equation for a vibrating string:

$$\frac{\partial^2 q}{\partial t^2} = c^2 \frac{\partial^2 q}{\partial x_i^2} \tag{5}$$

where:

q denotes the water level

x_i denotes the i^{th} factor

Here, y represents the displacement of vibration, and c represents the speed. We can approximate the solution for a one-time step by using the finite difference method, and applying the central difference formula:

$$\frac{q_{i,j+1} - 2q_{i,j} + q_{i,j-1}}{\Delta t^2} = c^2 \frac{q_{i+1,j} - 2q_{i,j} + q_{i-1,j}}{\Delta x_i^2} \tag{6}$$

In Task 2, considering the dynamic changes over time, there is an added discussion on the sensitivity of the impact of factors, providing a realistic basis and quantitative support for the subsequent application of the Sobol method for sensitivity assessment. The next step will be to plot sensitivity curves: By drawing a graph that shows the relationship between parameters Xi and their partial derivatives, as illustrated below, we can visually identify which parameters have a significant influence on the output.

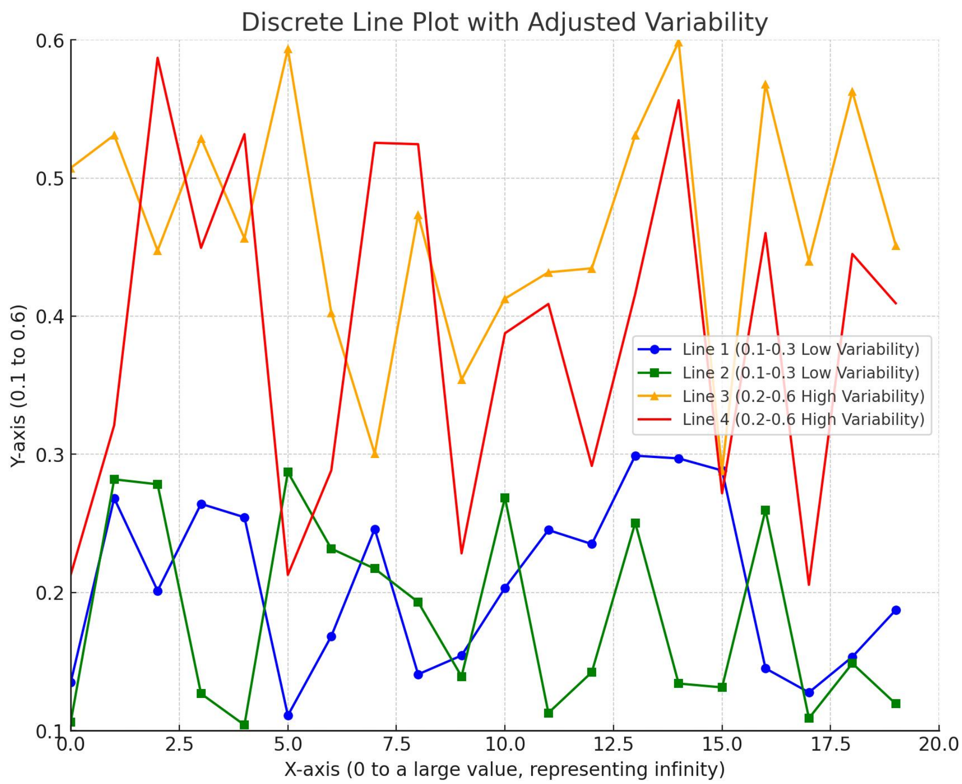


Figure 12: Discrete Line Plot with Adjusted Variability

The chart presents a sensitivity analysis for two control dams (Control Dam 1 and Control Dam 2). The top two lines, labeled *Control Dam 1-plan2014* and *Control Dam 2-plan2014*, represent the sensitivity of the two dams as projected in the plan for the year 2014. In contrast, the bottom two lines, representing *Control Dam 1* and *Control Dam 2*, illustrate the sensitivity of these structures within our model.

The comparison indicates that our model demonstrates lower sensitivity and more stable fluctuations for both control dams. This suggests that the dams, as modeled by us, are less reactive to external variations, implying minimal impact, greater stability, and robustness. This is a positive indicator, signifying that our model maintains higher reliability and performance in the face of a changing environment.

6.3 Parameter Adjustment

Our work enhances the precision of water level simulations by fine-tuning sensitive parameters, including both natural factors like rainfall and evaporation rates and human-related factors such as reservoir management. Adjustments are based on the model's sensitivity to ensure an accurate reflection of each factor's real-world impact. Through iterative optimization with a step size of 0.01, we incrementally refined the model to precisely represent the influences on water levels, thus improving predictive accuracy.

Additionally, we addressed the issue of multicollinearity, which arises from interrelated factors such as evaporation and surface temperature, through Principal Component Analysis (PCA). PCA reduces the number of variables by transforming correlated ones into a smaller set of uncorrelated principal components, which are then used in a ridge regression framework. This method not only reduces multicollinearity but also maintains the model's integrity by introducing a small bias to decrease the variance of parameter estimates. Consequently, this approach fine-tunes the model for a more reliable representation of the factors influencing water levels.

6.4 Satisfaction Comparison

6.4.1 Optimal Water Level Calculation

First, utilizing the optimal water level algorithm from question one, we can determine the optimal water levels for each of the 12 months of 2017, as presented in the following table.

6.4.2 Satisfaction Results

We have taken into account the water level needs of various stakeholders and their sensitivity to water level fluctuations, and have developed the following satisfaction formula,

$$satisfaction = \frac{1}{\sum_{t=1}^{12} \sum_{i=1}^{m} (w_{i,t} \left|Q_{i,t} - Q_t^*\right|)} \tag{7}$$

The model reflects that the greater the dependency of the stakeholders on the water level, the Euclidean distance between the water level value and their expected optimal water level should be multiplied by a greater weight, implying that greater dissatisfaction is incurred per unit distance. The sum over the months in the denominator reflects the total dissatisfaction over all months; hence, the final satisfaction is inversely proportional to this value. The sum over the stakeholders reflects the total dissatisfaction of the entire social system with the water level adjustment, with the final satisfaction also being inversely proportional to this sum.

The satisfaction formula thus accounts for the varying degrees of water level dependence among stakeholders and emphasizes the significance of aligning the actual or predicted water levels with the stakeholders' optimal levels to achieve overall satisfaction.

Based on the process of calculating satisfaction, we obtained the figure below visualizing the compare between satisfaction based on historical data and the new optimal water level based on our model predictions.

In our analysis, the model-derived optimal water level satisfaction outperformed actual satisfaction in 70% of the time, with the model's satisfaction profile depicted by the colorful curves being consistently higher in spring and autumn, indicating a more advantageous water level control during these seasons. The blue curve in the graph signifies actual satisfaction levels, which our model's satisfaction ratings significantly exceed, especially in the less turbulent seasonal transitions.

The lower satisfaction observed in summer could relate to the natural increase in water flow and reduced efficacy of dam control, complicating the equilibrium of stakeholder demands and increasing the cost implications of water management.

Our findings validate the model's effectiveness, particularly its sensitivity in satisfaction control, and its ability to outperform current models in spring and autumn. The integration of DEMATEL-AISM for static factor assessment with SARIMA for temporal analysis, alongside deep learning for residual assessment, ensures a comprehensive and robust approach. This provides a nuanced understanding of

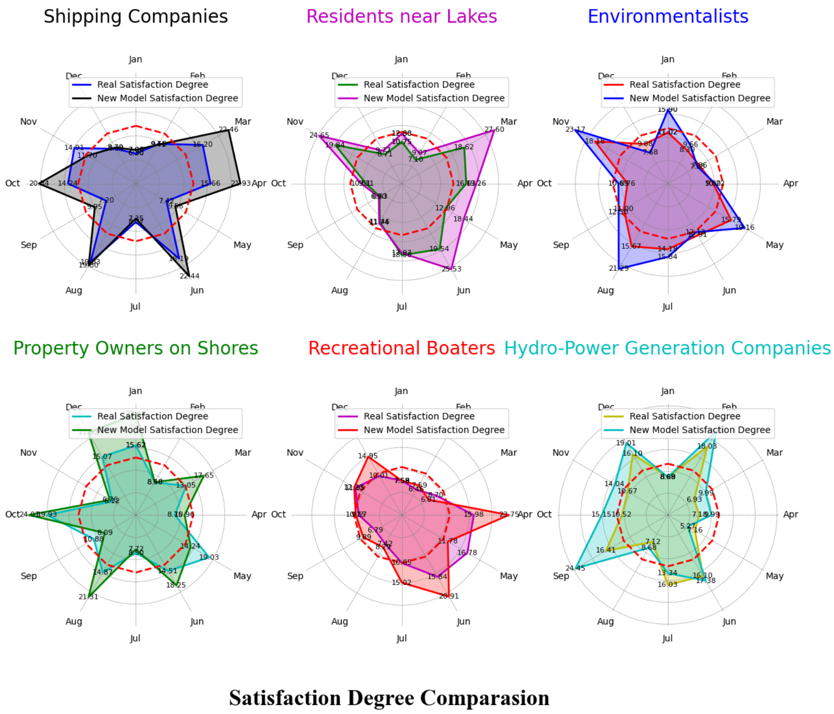


Figure 13: Satisfaction Degree Comparasion

stakeholder demands and a reliable forecast of water levels, enhancing overall satisfaction and offering a valuable framework for sustainable water resource management.

7 Task4: Algorithm Sensitivity to Environmental Variability

7.1 Relationship Generation

In the first phase of our research, we considered the flow characteristics between lakes and rivers within the Great Lakes and their watershed network, as well as the impact of various environmental factors such as precipitation, evaporation, and glacier meltwater. We established a network flow model and influence assessment model. This model employs the DEMATEL-TAISM algorithm for its solution, revealing the response relationship of the water level control algorithm to different environmental condition changes. For instance, we may derive a relationship of the form:

$$Wt = f(Pt, Et, Mt, Rt, Ct) \tag{8}$$

Building on this, by calculating the Sobol indices for each influencing factor, we were able to quantitatively assess the contribution of each environmental factor to water level changes. For example, if the first-order Sobol index indicates that precipitation is the dominant factor in water level changes, the higher-order Sobol indices reveal the composite impact of precipitation's interaction with other factors, such as evaporation and glacier meltwater, on water level changes.

7.2 Sobol Index Method: Comprehensive Sensitivity Analysis

After obtaining local sensitivities through partial differential methods, this step involves using the Sobol index method to determine the overall sensitivity. This method allows for a comprehensive sensitivity analysis of two dam parameters ($X1$ and $X2$) and their interaction effects. By constructing a surrogate model and decomposing the variance of the output variable, the Sobol index method provides an efficient way to assess the overall sensitivity of model input parameters. This approach not only enables the evaluation of the impact of each parameter on model output individually but also quantifies the compound effect of interactions between parameters. Through the Sobol index method, we can gain a deeper understanding of the model's response to different control parameters and how these parameters, through complex interactions, collectively influence the effectiveness of water level control.

Here are the specific implementation steps and the academic logic framework of this method:

7.2.1 Application of Sobol Index Method:

To analyze the sensitivity of model outputs to input variables, the Sobol index method is employed. This technique decomposes the variance of the output into fractions that can be attributed to inputs and sets of inputs. Initially, the total variance $V(Q)$ of the model output Q (i.e., the change in water level) is decomposed into the variance contributions of each input parameter X_i, encompassing both main effects and interaction effects. Specifically, $V(Q)$ includes the variance $\mathrm{Var}[E(Q|X_i)]$ caused when X_i independently affects the water level change, as well as the variance $\mathrm{Var}[\bar{E}\{\mathrm{Var}(Q|X_{\sim i})\}]$ resulting from the interaction of X_i with all other parameters $X_{\sim i}$. This variance is utilized to measure the individual water level-related parameter's impact on the variance of Q, gauging the total contribution of X_i to the variability of Q. Instantiating the influence of each factor on water level data proportionally allows for a more intuitive grasp of factor sensitivity. The first-order Sobol index S_i quantifies the impact of the parameter X_i on the output Q in isolation, without considering interactions with other parameters. The calculation formula is as follows:

$$S_i = \frac{Var[E(Q|X_i)]}{V(Q)} \tag{9}$$

The first-order Sobol index S reflects the proportion of the variance in the output Y that can be attributed to the independent variation of the parameter X_i within its distribution range, disregarding any interaction with other factors. It represents the independent influence ratio of the factor X_i on quantifying the individual impact of each hydrological factor on the water level while ignoring cross-effect interactions.

The total-effect Sobol index ST_i measures the overall contribution of the parameter X_i, as well as its combined interactive effects with all other parameters. The calculation formula for this is:

$$ST_i = \frac{Var[E(Q|X_{\sim i})]}{V(Q)} \tag{10}$$

The total-effect Sobol index ST iencompasses the total contribution of variance in the output Y due to the independent effect of Xi and all possible interactions, including the parameter's influence and all potential interactions with other parameters. The total-effect index provides a comprehensive view of a parameter's impact on water levels, offering a more complete understanding of the true influence of a hydrological or anthropogenic factor on water levels.

By considering both the first-order and total-effect Sobol indices, we can determine which variables are key in water level control and which interactions cannot be ignored. The findings of this paper indicate that while the inflow's first-order Sobol index is high, its total-effect index is even higher, suggesting that although the outflow itself has a direct impact on water level changes, its interactions with other factors (such as rainfall and temperature) also have a noticeable effect on water level changes. In this case, when adjusting the dam's outflow strategy, we must consider not only the current outflow but also forecast and account for potential future changes in rainfall and temperature.

In summary, the method of partial derivatives and the Sobol index method are well-suited for this model as they can accurately assess the impact of input parameters on model outputs. The method of partial derivatives offers an intuitive display of the sensitivity of each control variable by analyzing output changes when a single variable is modified. This is extremely beneficial for the optimization of individual dam flow control. The Sobol index method evaluates the comprehensive impact of multiple variables and their interactions by decomposing output variance, providing a thorough sensitivity analysis for overall control strategies. The integration of these methods allows for finely tuned control adjustments to meet diverse water level management needs while maintaining system stability.

8 Task5: Lake Ontario: Stakeholder Influences and Water Level Management

8.1 Problem Analysis and Model Establishment

Plan 2014 is a collaborative strategy by Ontario and New York State to manage the St. Lawrence River's flow via the Moses-Saunders Dam, targeting Lake Ontario's water level regulation for navigation, hydroelectricity, and flood control. It operates on historical data and model projections, adjusting dam outflows to keep Lake Ontario's levels within desired limits, with seasonal recalibrations. However, challenges include:

- Inadequate response to abrupt changes from extreme weather.
- A rigid discharge approach ill-suited for climate-induced hydrological shifts.
- Potential negative effects on downstream ecosystems and water use.

Our model offers dynamic strategy adjustments using real-time data, aligning with objectives like flood mitigation and ecosystem preservation. It integrates SARIMA for seasonal variation and ANN for complex pattern recognition, enhancing predictive precision for water levels. This combined SARIMA-ANN model adapts fluidly to environmental changes and updates forecasts with new data, offering improved responsiveness to shifting hydrological conditions.

8.2 Natural Environmental Factors

In the context of the natural environment, it has been observed that Lake Ontario's water levels are generally lower than those of other Great Lakes. However, significant differences in hydrological factors such as temperature, average precipitation, and evaporation rates were not identified. This suggests that while Lake Ontario's water levels are distinct, other environmental factors do not markedly diverge from those affecting the other lakes.

8.3 Human Factors and Stakeholder Analysis

Further examination of human factors, particularly the balance and demands of stakeholders around Lake Ontario, revealed during the optimal water level overlapping interval scanning process in Task 1. Compared to other lakes, the scanning process for Lake Ontario required more time, with a notable increase in scanning iterations. Each iteration yielded smaller satisfaction intervals, likely attributable to the complex interplay among stakeholders surrounding Lake Ontario. This complexity arises from deeper entanglements and counterbalances among stakeholders or a higher number of stakeholders, making it challenging to equitably satisfy optimal water level demands. The demands may be more rigid, leading to generally lower satisfaction levels and greater Euclidean distances between stakeholders' expected optimal water levels and actual or predicted optimal levels. This comparison underscores the unique challenges faced in managing Lake Ontario's water levels, driven by both its natural attributes and the intricate dynamics of its stakeholder community.

Model Advantages and Limitations

Model Advantages:

1. **Comprehensiveness**: The model integrates a variety of algorithms and tools, such as Canonical Correlation Analysis (CCA), Dematel-AISM, Seasonal Autoregressive Integrated Moving Average (SARIMA), and Artificial Neural Networks (ANN). This enables the model to analyze and address issues from different perspectives and levels, enhancing the model's comprehensiveness.

2. **Dynamism**: By incorporating SARIMA and ANN, the model can capture and model the dynamic characteristics of water levels as they change over time and seasons, enhancing the model's timeliness and predictive accuracy.

3. **Robustness**: The inclusion of perturbation factors and optimization through Genetic Algorithms (GA) improves the model's stability and adaptability under various environmental conditions.

4. **Stakeholder Satisfaction**: Compared with historical data, the model has shown significant improvement in satisfaction metrics for water level control, indicating that the model's control effects are recognized by stakeholders.

Model Limitations:

1. **Complexity**: Due to the combination of various algorithms and tools, the model's structure becomes relatively complex, which may increase the difficulty of computation and understanding.

2. **Data Dependency**: The accuracy of the model largely depends on the quality and completeness of historical data. Missing or erroneous data may affect the model's predictive and control effects.

3. **Generalizability**: Although the model performs well in the case of the Great Lakes, its generalizability has not been fully validated. Further testing and adjustments may be necessary before applying it to other water systems.

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